CHAPTER 9

Permutation and Combination

Exercise

1. The number of ways in which a team of eleven players can be selected from 22 players including 2 of them and excluding 4 of them is
   (a) $^{16}C_{11}$ (b) $^{16}C_5$
   (c) $^{16}C_9$ (d) $^{20}C_9$
2. The number of permutations of all the letters of the word 'EXERCISES' is
   (a) 60480 (b) 30240
   (c) 10080 (d) None of these
3. The total number of selections of fruits which can be made from 3 bananas, 4 apples and 2 oranges is
   (a) 39 (b) 315
   (c) 512 (d) 59
4. How many 10 digits numbers can be written by using the digits 1 and 2 ?
   (a) $^{10}C_1 + {}^9C_2$ (b) 2^{10}
   (c) $^{10}C_2$ (d) 10!
5. How many four digit numbers divisible by 10 can be formed using 1, 5, 0, 6, 7 without repetition of digits?
   (a) 24 (b) 36
   (c) 44 (d) 64
6. A meeting is to be addressed by 5 speakers A, B, C, D and E. In how many ways can the speakers be ordered, if B must not precede A (immediately or otherwise) ?
   (a) 120 (b) 24
   (c) 60 (d) $5^4 \times 4$
7. The number of straight lines that can be formed out of 10 points, of which 7 are collinear, are
   (a) 26 (b) 21
   (c) 25 (d) None of these
8. There are 18 points in a plane such that no three of them are in the same line except five points which are collinear. The number of triangles formed by these points is
   (a) 805 (b) 806
   (c) 816 (d) None of these
9. Everybody in a room shakes hands with everybody else. The total number of hand shakes is 66. The total number of persons in the room is
   (a) 11 (b) 12
   (c) 13 (d) 14
10. On the occasion of Dipawali festival each student of a class sends greeting cards to the others. If there are 20 students in the class, then the total number of greeting cards exchanged by the students is
    (a) $^{20}C_2$ (b) $2 . {}^{20}C_2$
    (c) $2 . {}^{20}P_2$ (d) None of these
11. The total number of words which can be formed out of the letters *a, b, c, d, e, f* taken 3 together, such that each word contains at least one vowel, is
    (a) 72 (b) 48
    (c) 96 (d) None of the above
12. The number of ways in which 8 different flowers can be strung to form a garland so that 4 particular flowers are never separated is
    (a) $4! \cdot 4!$ (b) $\frac{8!}{4!}$
    (c) 288 (d) None of these
13. A box contains two white balls, three black balls and four red balls. In how many ways can three balls be drawn from the box if at least one black ball is to be included in the draw ?
    (a) 129 (b) 84
    (c) 64 (d) None of these
14. A polygon has 170 diagonals. How many sides will it have ?
    (a) 12 (b) 17
    (c) 20 (d) 25

15. A five digit number divisible by 3 is to be formed using the numerals 0, 1, 2, 3, 4 and 5, without repetition. The total number of ways this can be done is
(a) 216 (b) 240
(c) 600 (d) 3125

16. All possible four digit numbers are formed using the digits 0, 1, 2, 3 so that no number has repeated digits. The number of even numbers among them is
(a) 9 (b) 18
(c) 10 (d) None of these

17. The letters of the word RANDOM are written in all possible orders and these words are written out as in a dictionary then the rank of the word RANDOM is
(a) 614 (b) 615
(c) 613 (d) 616

18. If all permutations of the letters of the word AGAIN are arranged in dictionary, then fiftieth word is
(a) NAAGI (b) NAGAI
(c) NAAIG (d) NAIAG

19. The number of all four digit numbers which are divisible by 4 that can be formed from the digits 1, 2, 3, 4 and 5 is
(a) 125 (b) 30
(c) 95 (d) None of these

20. The number of all five digit numbers which are divisible by 4 that can be formed from the digits 0, 1, 2, 3, 4 (without repetition) is
(a) 36 (b) 30
(c) 34 (d) None of these

21. The total number of numbers greater than 1000 but not greater than 4000 that can be formed with the digits 0, 1, 2, 3, 4 when the repetition of digits allowed is
(a) 375 (b) 374
(c) 376 (d) None of these

22. There are 5 letters and 5 directed envelopes. The number of ways in which all the letters can be put in wrong envelope is
(a) 119 (b) 44
(c) 59 (d) 40

23. A committee of 5 is to be formed from 9 ladies and 8 men. If the committee commands a lady majority, then the number of ways this can be done is
(a) 2352 (b) 1008
(c) 3360 (d) 3486

24. The total number of ways in which 12 persons can be divided into three groups of 4 persons each is
(a) $\dfrac{12!}{(3!)^3 4!}$ (b) $\dfrac{12!}{(4!)^3}$
(c) $\dfrac{12!}{(4!)^3 3!}$ (d) $\dfrac{12!}{(3!)^4}$

25. The total number of ways in which 4 boys and 4 girls can form a line, with boys and girls alternating, is
(a) $(4!)^2$ (b) 8!
(c) $2(4!)^2$ (d) $4!\,.\,{}^5P_4$

26. The number of different words (eight letter words) ending and beginning with a consonant which can be made out of the letters of the word 'EQUATION' is
(a) 5200 (b) 4320
(c) 3000 (d) 2160

27. In a football championship, 153 matches were played. Every team played one match with each other. The number of teams participating in the championship is
(a) 17 (b) 18
(c) 9 (d) None of these

28. Seven women and seven men are to sit round a circular table such that there is a man on either side of every women. The number of seating arrangements is
(a) $(7!)^2$ (b) $(6!)^2$
(c) $6! \times 7!$ (d) 7!

29. There are $(n + 1)$ white and $(n + 1)$ black balls each set numbered 1 to $n + 1$. The number of ways in which the balls can be arranged in a row so that the adjacent balls are of different colours is
(a) $(2n + 2)!$ (b) $(2n + 2)! \times 2$
(c) $(n + 1)! \times 2$ (d) $2\{(n + 1)!\}^2$

30. A group consists of 5 men and 5 women. If the number of different five persons committees containing k men and $(5 - k)$ women is 100, what is the value of k ?
(a) 2 only (b) 3 only
(c) 2 or 3 (d) 4 only

31. In how many ways can the letters of the word 'CABLE' be arranged, so that the vowels should always occupy odd positions ?
(a) 12 (b) 18
(c) 24 (d) 36

32. If ${}^nC_{r-1} = 36$, ${}^nC_r = 84$, ${}^nC_{r+1} = 126$, then (n, r) is equal to
(a) (8, 4) (b) (7, 5)
(c) (9, 3) (d) None of these

33. The number of different words that can be formed from the letters of the word 'TRIANGLE' so that no two vowels are together is
(a) 7200 (b) 36000
(c) 14400 (d) 1240

34. The number of arrangements that can be made out of the letters of the word 'SUCCESS' so that all S's do not come together is
(a) 60 (b) 120
(c) 360 (d) 420

35. How many different nine digit numbers can be formed from the number 223355888 by rearranging its digits so that the odd digits occupy even positions ?
(a) 16 (b) 36
(c) 60 (d) 180

36. 20 persons were invited for a party. The number of ways in which they and the host can be seated at a circular table such that two particular persons be seated on either side of the host is
(a) 20! (b) 19!
(c) 2 (18!) (d) 18!

37. The number of ways that 5 students can be made to sit in a row so that the tallest and shortest may not come together is
(a) 48 (b) 24
(c) 72 (d) 120

38. Eleven animals of a circus have to be placed in eleven cages, one in each cage. If 4 of the cages are too small for 6 of the animals, the number of ways of caging the animals, is
(a) 7!.5! (b) 4!.6!
(c) 6!.6! (d) None of these

39. The number of ways in which the following prizes be given to a class of 20 boys, first and second Math ematics, first and second Physics, first Chemistry and first English is
(a) $20^4 \times 19^2$ (b) $20^3 \times 19^3$
(c) $20^2 \times 19^4$ (d) None of these

40. A code word consists of three letters of the English alphabet followed by two digits of the decimal system. If neither letter nor digit is repeated in any code word, then the total numbers of code words are
(a) 1404000 (b) 16848000
(c) 2808000 (d) None of these

41. A candidate is required to answer 7 out of 15 questions which are divided into three groups A, B, C each containing 4, 5, 6 questions respectively. He is required to select at least 2 questions from each group. He can make up his choices in
(a) 1200 ways (b) 2700 ways
(c) 2000 ways (d) None of these

42. Number of different four digit numbers that may be formed using each of the digits 1, 2, 3, 4, 5, 6, 7 and 8 only once so that the number contains 4 is
(a) ${}^8P_4$ (b) ${}^8C_4$
(c) ${}^7C_3.4!$ (d) None of these

43. What is the sum of all the digits at unit place formed by 3, 4, 5 and 6 taken all at a time ?
(a) 108 (b) 72
(c) 124 (d) None of these

44. The total number of arrangements which can be made out of the letters of the word ALGEBRA without altering the relative positions of vowels and consonants is
(a) $\frac{7!}{2!}$ (b) $\frac{7}{2!\,5!}$
(c) 4! 3! (d) $\frac{4!\,3!}{2}$

45. What is the number of ways in which 3 holiday travel tickets are to be given to 10 employees of an organisation, if each employee is eligible for any one or more of the tickets? **[NDA-I 2016]**
(a) 60 (b) 120
(c) 500 (d) 1000

46. What is the number of four digit decimal numbers (< 1) in which no digit is repeated? **[NDA-I 2016]**
(a) 3024 (b) 4536
(c) 5040 (d) None of these

47. What is the number of different messages that can be represented by three 0's and two 1's? **[NDA-I 2016]**
(a) 10 (b) 9
(c) 8 (d) 7

48. What is the number of odd integers between 1000 and 9999 with no digit repeated? **[NDA-II 2016]**
(a) 2100 (b) 2120
(c) 2240 (d) 3331

49. Out of 15 points in a plane, n points are in the same straight line. 445 triangles can be formed by joining these points. What is the value of n? **[NDA-II 2016]**
(a) 3 (b) 4
(c) 5 (d) 6

50. A five digit number divisible by 3 is to be formed using the digits 0, 1, 2, 3 and 4 without repetition of digits. What is the number of ways this can be done? **[NDA-II 2016]**
(a) 96
(b) 48
(c) 32
(d) No number can be formed

ANSWERS

1.	(c)	**2.**	(b)	**3.**	(d)	**4.**	(b)	**5.**	(a)	**6.**	(c)	**7.**	(c)	**8.**	(b)	**9.**	(b)	**10.**	(b)
11.	(c)	**12.**	(a)	**13.**	(c)	**14.**	(c)	**15.**	(a)	**16.**	(c)	**17.**	(a)	**18.**	(c)	**19.**	(a)	**20.**	(b)
21.	(a)	**22.**	(b)	**23.**	(d)	**24.**	(c)	**25.**	(c)	**26.**	(b)	**27.**	(b)	**28.**	(c)	**29.**	(d)	**30.**	(c)
31.	(b)	**32.**	(c)	**33.**	(c)	**34.**	(c)	**35.**	(c)	**36.**	(c)	**37.**	(c)	**38.**	(a)	**39.**	(a)	**40.**	(a)
41.	(b)	**42.**	(c)	**43.**	(a)	**44.**	(d)	**45.**	(d)	**46.**	(c)	**47.**	(a)	**48.**	(c)	**49.**	(c)	**50.**	(d)

Explanations

1. (c) Team of 11 is to be formed form 22 players including 2 and excluding 4 of ways
$= {}^{22-2-4}C_{11-2} = {}^{16}C_9$

2. (b) Total number of permutations
$$= \frac{9!}{3! \times 2!} = 30240$$

3. (d) Total number of selections
$= (3 + 1)(4 + 1)(2 + 1) - 1 = 59$

4. (b) Total number having 10 digits $= 2^{10}$

5. (a) If a four digit number is divisible by 10, then last digit has to be filled by 0.
Then, remaining 4 numbers {1, 5, 6, 7} can be arranged in ${}^4P_3$ ways.
Hence, total arrangements $= {}^4P_3 \times 1$
$$= \frac{4!}{(4-3)!} \times 1 = 4 \times 3 \times 2 = 24$$

6. (c) If B must not precede A (immediately or otherwise) then B must follow A.
i.e., A can have the following places.
$\boxed{A}$ _ _ _ _ = 4! = 24
_ $\boxed{A}$ _ _ _ = 3 × 3! = 18
_ _ $\boxed{A}$ _ _ = 3 × 2 × 2 × 1 = 12
_ _ _ $\boxed{A}$ _ = 3! = 6
∴ Required number of ways
= 24 + 18 + 12 + 6 = 60

7. (c) Number of straight lines $= {}^{10}C_2 - {}^7C_2 + 1 = 25$

8. (b) Number of triangles $= {}^{18}C_3 - {}^5C_3 = 806$

9. (b) Total number of handshakes
${}^nC_2 = 66$
$$\Rightarrow \frac{n(n-1)}{2!} = 66$$
$\Rightarrow n(n-1) = 132$
$\Rightarrow n = 12$
So, total number of persons = 12

10. (b) On Dipawali occasian, each student gives greeting card to other.
So, total number of cards exchanged by students
$= 2 \,.\, {}^{20}C_2$

11. (c) There are 2 vowels and 4 consonants.
So, required ways
$= ({}^2C_1 \times {}^4C_2 + {}^2C_2 \times {}^4C_1) \times 3!$
$= 16 \times 6 = 96$

12. (a) 4 particular flowers are never separated from a garland of 8 flowers. So, required ways
$= (8 - 4 + 1)! \times 4! = 4! \times 4!$

13. (c) Three balls are to be selected from 2 white, 3 black and 4 red balls such that atleast 1 black ball is to be included.
So, required ways
$= {}^3C_1 \times {}^6C_2 + {}^3C_2 \times {}^6C_1 + {}^3C_3 = 64$

14. (c) Let polygon has n sides.
$$\text{So, diagonals} = \frac{n(n-3)}{2}$$
$$\Rightarrow \frac{n(n-3)}{2} = 170$$
$\Rightarrow n(n-3) = 340$
$\Rightarrow n = 20$

15. (a) For a number divisible by 3, the sum of digits of a number should be divisible by 3.
0 + 1 + 2 + 3 + 4 + 5 = 15
But we have to form a number of 5 digits.
So, either 0 should be left or 3 should be left.
Hence, required ways
= 5 × 4 × 3 × 2 × 1 + 4 × 4 × 3 × 2 × 1
= 120 + 96 = 216

16. (c) For even number last digit should be divisible by 2.
— — — $\boxed{0}$ = 3 × 2 × 1 = 6
— — — $\boxed{2}$ = 2 × 2 × 1 = 4
So, total numbers = 10

17. (a) A — — — — — = 5! = 120
D — — — — — = 5! = 120
M — — — — — = 5! = 120
N — — — — — = 5! = 120
O — — — — — = 5! = 120
R A D — — — = 3! = 6
R A M — — — = 3! = 6
R A N D M O = 1
R A N D O M = 1
So, rank of RANDOM = 614

18. (c) A – – – – = 4! = 24
$$G - - - - = \frac{4!}{2!} = 12$$
$$I - - - - = \frac{4!}{2!} = 12$$
49th word = N A A G I
50th word = N A A I G

19. (a) For the number divisible by 4, last two digits should be divisible by 4.

— — $\boxed{1}\boxed{2}$ = 5 × 5 = 25

— — $\boxed{2}\boxed{4}$ = 5 × 5 = 25

— — $\boxed{4}\boxed{4}$ = 5 × 5 = 25

— — $\boxed{3}\boxed{2}$ = 5 × 5 = 25

— — $\boxed{5}\boxed{2}$ = 5 × 5 = 25

Total numbers = 125

20. (b) — — — $\boxed{0}\boxed{4}$ = 3 × 2 × 1 = 6

— — — $\boxed{4}\boxed{0}$ = 3 × 2 × 1 = 6

— — — $\boxed{1}\boxed{2}$ = 2 × 2 × 1 = 4

— — — $\boxed{2}\boxed{0}$ = 3 × 2 × 1 = 6

— — — $\boxed{2}\boxed{4}$ = 2 × 2 × 1 = 4

— — — $\boxed{3}\boxed{2}$ = 2 × 2 × 1 = 4

Total numbers = 30

21. (a) Numbers greater than 1000 and lesser than 4000.

$\boxed{1}$ — — — = 5 × 5 × 5 – 1 = 124

$\boxed{2}$ — — — = 5 × 5 × 5 = 125

$\boxed{3}$ — — — = 5 × 5 × 5 = 125

$\boxed{4}\boxed{0}\boxed{0}\boxed{0}$ = 1 × 1 × 1 = 1

So, total numbers = 375

22. (b) Required number of ways

$$= 5!\left[1+\frac{1}{1!}+\frac{1}{2!}-\frac{1}{3!}+\frac{1}{4!}-\frac{1}{5!}\right] = 44$$

23. (d) 5 members committee is to be formed among 9 ladies and 8 men such that ladies are in majority.

Number of ways = 3 ladies and 2 men or 4 ladies and 1men or 5 ladies

$= {}^9C_3 \times {}^8C_2 + {}^9C_4 \times {}^8C_1 + {}^9C_5$

= 3486

24. (c) Total number of ways of dividing 12 persons into 3 groups of 4 persons $= \dfrac{(12)!}{(4!)^3 \times 3!}$

25. (c) Total number of ways of arranging boys and girls alternating = 2(4! × 4!) = 2 × $(4!)^2$

26. (b) In word 'EQUATION' there are 5 vowels and 3 consonants. Word should end and start with consonant. So, there are 3 and 2 choices for first and last position.

Hence, total words

= 3 × 6 × 5 × 4 × 3 × 2 × 2

= 4320

27. (b) Let number of teams = n

So, ${}^nC_2 = 153$

$$\Rightarrow \frac{n(n-1)}{2} = 153 \Rightarrow n = 18$$

28. (c) Number of ways of arranging 7 men around the circular table = (7 – 1)! = 6!

There are 7 places for women.

Hence, required number of ways = 6! × 7!

29. (d) Required number of ways of arranging white and black balls = $2\{(n+1)!\}^2$

30. (c) ${}^5C_k \times {}^5C_{5-k} = 100$

$$\Rightarrow \frac{5!}{k!(5-k)!} \times \frac{5!}{k!(5-k)!} = 100$$

$$\Rightarrow \left(\frac{5!}{k!(5-k)!}\right)^2 = 100$$

$\Rightarrow k = 2$ or 3.

31. (b) Required number of ways = ${}^3P_2 \times 3! = 18$

32. (c) ${}^nC_{r-1} = 36, = {}^nC_r = 84, {}^nC_{r+1} = 126$

$$\Rightarrow \frac{{}^nC_r}{{}^nC_{r-1}} = \frac{84}{36} = \frac{7}{3} \text{ and}$$

$$\Rightarrow \frac{{}^nC_{r+1}}{{}^nC_r} = \frac{126}{84} = \frac{3}{2}$$

$$\Rightarrow \frac{n-r+1}{r} = \frac{7}{3} \text{ and } \frac{n-r}{r+1} = \frac{3}{2}$$

$\Rightarrow 10r - 3n = 3$ and $2n - 5r = 3$

On solving, $n = 9$ and $r = 3$

33. (c) In the word 'TRIANGLE', there are 3 vowels and 5 consonants.

So, total arrangements when no vowels are together

$= {}^5P_5 \times {}^6P_3 = 120 \times 120 = 14400$

34. (c) Total arrangements of the letters of word 'SUCCESS'

$$= \frac{7!}{2!\,3!} = 420$$

Arrangements when all S's come together

$$= \frac{(7-3+1)!}{2!} = 60$$

So, required arrangements when all S's not come together = 420 – 60 = 360

35. (c) There are 4 odd digits and 4 even positions.

It can be arrange in $\dfrac{4!}{2!\,2!} = 6$ ways

Remaining 5 even digits are to be arranged in 5 places. It can be done in $\dfrac{5!}{2!\,3!} = 10$ ways

Hence, required arrangements = 6 × 10 = 60

36. (c) 20 guests and 1 host = 21 persons
Let us consider the host and 2 particular persons as one unit. So, we have 21 – 3 + 1 = 19 persons and numbers of arrangements = (19 – 1)! = 18!. But two persons on either sides of the host can be arranged in 2! ways.
So, total number of ways = 2 (18!)

37. (c) Total arrangements that 5 students sit in a row
= 5! = 120
Arrangements when two particular always come together = 4! × 2! = 48
Hence, number of ways when the tallest and the shortest not come together = 120 – 48 = 72

38. (a) Large animals = 6; Small animals = 5
Large cages = 7; Small cages = 4
6 large animals can be placed in 7 large cages and remaining 5 small animals can be place in remaining 5 cages.
So, total arrangements = ${}^7P_6 \times {}^5P_5 = 7! \times 5!$

39. (a) Number of ways = $20^4 \times 19^2$

40. (a) There are 26 alphabets and 10 digits. We have to arrange 3 alphabets followed by 2 digits.
So, total arrangements = ${}^{26}P_3 \times {}^{10}P_2 = 1404000$

41. (b) (2A 2B 3C) + (2A 3B 2C) + (3A 2B 2C)
$= ({}^4C_2 \,.\, {}^5C_2 \,.\, {}^6C_3) + ({}^4C_2 \,.\, {}^5C_3 \,.\, {}^6C_2) + ({}^4C_3 \,.\, {}^5C_2 \,.\, {}^6C_2)$
= 1200 + 900 + 600 = 2700

42. (c) ∵ 4 is must, so we have to select 3 numbers out of remaining 7 numbers. It can be done in ${}^7C_3$ ways.
Hence, total numbers = ${}^7C_3 \times 4!$

43. (a) If 3 is fixed in the unit place then remaining three digits can be arranged in 3! = 6 ways.
Similarly, each other digit will be in unit place 6 times.
So, sum of digits in unit place
= 6 (3 + 4 + 5 + 6) = 108

44. (d) All the consonants can be arranged in 4 places.
So, arrangements of consonants = 4! = 24
Similarly, vowels can be arranged in 3 places but 2 vowels are identical. So, arrangement of vowels
$= \frac{3!}{2!} = 3$
Hence, total number of ways = 24 × 3 = 72

45. (d) Since each employee is eligible to get any one or more of the tickets, i.e., there are 10 choices for every ticket. So, total number of ways of giving tickets = 10 × 10 × 10 = 1000

46. (c) Total ways of forming four digit numbers less than 1 = ${}^{10}P_4 = 5040$
Four digit numbers less than 1 having 0 at last place = ${}^9P_3 = 504$
Hence, required numbers = 5040 – 504 = 4536

47. (a) Here are three 0's and two 1's.
So, we have to arrange 5 items out of which 3 are identical and 2 are identical.
So, total different messages = $\frac{5!}{2!\,3!} = 10$

48. (c) □□□□□□
For odd integers we can place 1, 3, 5, 7, 9 at the unit digit.
So, total choices for unit digit = 5
∵ No digit is to be repeated and 0 cannot come at first place. So, we have 8, 8 and 7 choices for first, second and third places respectively.
So, number of odd integers = 8 × 8 × 7 × 5
= 2240

49. (c) Out of 15 points, n are collinear.
So, total number of triangles = ${}^{15}C_3 - {}^nC_3$
$\Rightarrow {}^{15}C_3 - {}^nC_3 = 445$
$\Rightarrow \frac{15!}{3!\,12!} - {}^nC_3 = 445$
$\Rightarrow {}^nC_3 = 455 - 445 = 10$
$\Rightarrow n = 5$

50. (d) For a number to be divisible by 3, the sum of its digits should be divisible by 3.
Here, 0 + 1 + 2 + 3 + 4 = 10
It is not divisible by 3.
So, total number of ways = 0